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A STUDY OF TWO-LAYER GAS FLOW IN SUPERSONIC AXISYMMETRIC NOZZLES

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A STUDY OF TWO-LAYER GAS FLOW IN SUPERSONIC AXISYMMETRIC NOZZLES

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ABSTRACT. Two methods of calculating two-layer flow are described. The first is a generalization of the numerical method of determining the inverse problem [1] to the case of two-layer flows without taking mixing into consideration. The second is a method of characteristics for calculating two-layer flows in supersonic nozzles. Here, the usual method of characteristics is modified to calculate a point on the line separating layers having different adiabatic exponents, different total pressures and temperatures. This paper also presents the results of calculating a two-layer flow in nozzles with different adiabatic exponent and gas flowrate ratios in the layers.

1. In [1] calculations of super and subsonic irrotational isentropic flow for an ideal gas utilized a system of gas dynamic equations written with the variables ψ (current function) and x (Cartesian coordinates in the meridian plane). The flow field was defined as the result of numerical solution of an inverse problem in nozzle theory. With coordinates ψ, x it is convenient to carry out calculations of multilayer flows with distinct physical properties. Such calculations can be made in the framework of an ideal liquid without taking layer mixing into account. The total temperature, total pressure and adiabatic exponent may then be different in the layers. We shall designate the flow core parameters with subscript 1 and the parameters of /76*

*Numbers in the margin indicate the pagination in the original foreign text.

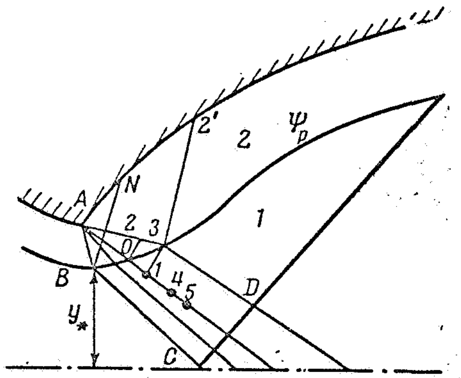

Figure 1.

the boundary layer with subscript 2 (Figure 1). If we set $\psi=\psi_p$ up to a certain current line, the gas will have the adiabatic exponent $k=k_1$ and a total pressure of p_{01}. Beginning at this current line, it will have the adiabatic exponent $k=k_2$ and a total pressure p_{02}. On the current line $\psi=\psi_p$, both flows should have identical velocity angles of inclination and identical static pressure values.

In this connection, at the current line $\psi=\psi_p$ the following relationships should be satisfied.

$$p_2=\left(\frac{k_2+1}{2}\right)^{k_2/(k_2-1)}\frac{1}{\sigma}\left[1-\frac{k_1-1}{k_1+1}w_1^2\right]^{k_1/(k_1-1)} \qquad \left(\sigma=\frac{p_{02}}{p_{01}}\right) \tag{1.1}$$

$$v_2=\frac{v_1}{w_1}\left[\frac{k_2+1}{k_2-1}-2\frac{p_2^{(k_2-1)/k_2}}{k_2-1}\right]^{1/2} \qquad (w=\sqrt{u^2+v^2}) \tag{1.2}$$

$$u_2=u_1\frac{v_2}{v_1}, \qquad \rho_2=p_2^{1/k_2} \tag{1.3}$$

Here u and v are projections of the velocity vector w on the x and y axes /77
of a Cartesian system of coordinates with respect to a — the critical speed of sound; p, ρ are the pressure and density in relation to the pressure and density for $w=a_*$; p_0 is the total pressure. After calculating the flow field by the difference Formulas (1.2)-(1.5) of [1] up to the current line with $\psi=\psi_p$ on this line Formulas (1.1)-(1.3) of this reference are used to determine the parameters with subscript 2 based on the known parameters with subscript 1. The calculation is continued using Formulas (1.2)-(1.5) of [1].

The results of calculating a two-layer flow with a velocity distribution on the axis (3.2) of [1] for $w_\infty=0.1$, $\overline{w}_\infty=1.9$, $1/b=3.5$ and $k_1=1.14$

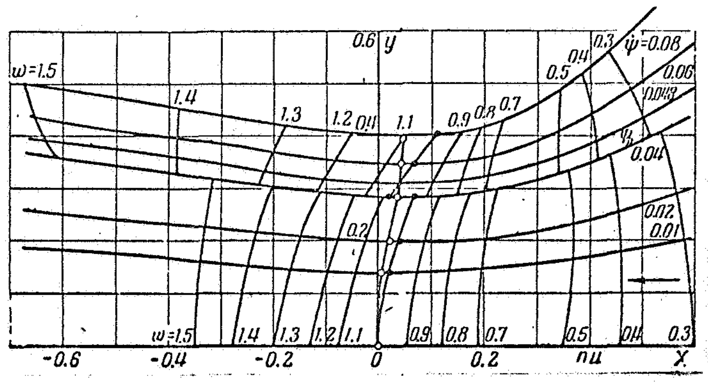

Figure 2.

$k_2 = 1.4$, $\sigma = 1$ and $\psi_p = 0.04$ are given in Figure 2, where we have shown the current line for the calculated flow ($\psi = \text{const}$), the line w = const, and the line $\theta = 0$ (light dots). According to Figure 2, in the second layer the sonic line w = 1 (dark dots) is located farther below the flow than in the first layer. From this figure, it also follows that the value of w on both sides of the separation boundary are quite close to each other in the subsonic region, but they become markedly different from the transsonic region on.

2. Let us consider supersonic two-layer flow in a nozzle with an angular point and a given profile. On the initial characteristic ABC, the flow parameters (Figure 1) are known. The calculations given below were made on the condition that the velocity vector on initial characteristic ABC was parallel to the x-axis, and the value β_{32} was assigned on characteristic AB. Moreover, it was assumed that $\beta = \sqrt{M^2 - 1}$ on characteristic AB was constant (M = 1.01) and equal to the value β_{32} on the line of separation in region 2. On characteristic BC, the quantity β was also assumed to be constant and equal to a value of β_{31} on the line of separation in region 1. Thus, a value of was found from the condition that the gas static pressures were equal on the line of separation, from which it follows that:

$$\beta_{32}=\left\{\frac{2}{k_2-1}\left[\sigma\left(\frac{k_1+1}{2}+\frac{k_1-1}{2}\beta_{31}^2\right)^{k_1/(k_1-1)}\right]^{(k_2-1)/k_2}-\frac{k_2+1}{k_2-1}\right\}^{1/2} \tag{2.1}$$

Since the values of β_{32} and β_{31} at point B are different, while the angles of inclination of velocity θ_{31} and θ_{32} coincide, the angle of inclination of characteristic AC at point B undergoes a break.

We may write the formulas for calculations by the method of characteristics of the point on the line of separation between the two layers. The computational formulas for points in the angular point field, points on the axis of symmetry and on the profile in each layer are considered in greater detail in [2]. Point 3, situated on the line of separation, is characterized by the parameters x, y, β_{32}, β_{31}, $\zeta = \operatorname{tg}\theta$, which need to be defined. To determine them, /78
we use the known points 0, 2, 4, 5 and the auxiliary point 1, located on the characteristic of the second family, whose location is not defined. To find the parameters at point 3, we use the following conditions: (1) point 3 lies at the intersection of the first family characteristics, passing through points 1 and 3, and the second family characteristics passing through points 2 and 3; (2) point 3 lies on the current line of the line of separation 03; (3) the static pressures at point 3 on both sides of the line of separation are equal; (4) point 1 lies on the second family characteristics in region 1. If the x coordinate for this point is known, the remaining parameters at the point are determined by quadratic interpolation from the known points 0, 4 and 5.

These conditions enable us to obtain six equations for the six unknown functions, some of which are found as a result by replacing the differential equations for direction and consistency along the first and second family characteristics by finite difference relationships [2]. Depending on the inclination of the characteristics of the first and second families in a calculation of a point on the separation line, as well as in the case of one-layer flow [2], we have four variants of formulas for computing the unknown quantities at a point on the line of separation. For the case when the inclinations of the first and second family characteristics are close to $^1/_2\pi$, the formulas have the form

$$x_3 = [x_2 + n^\circ(y_0 - y_2) - {}^1/_2(\zeta_3 + \zeta_0)x_0] / [1 - {}^1/_2 n^\circ(\zeta_3 + \zeta_0)] \tag{2.2}$$

$$y_3 = y_0 + {}^1/_2(\zeta_3 + \zeta_0)(x_3 - x_0) \tag{2.3}$$

$$x_1 = \frac{y_3(m - n^\circ) - x_2 + n^\circ y_2 + mn_0 x_0 - n_0 y_0}{mn_0 - 1} \tag{2.4}$$

$$K\beta_{31} + J\beta_{32} = \zeta_1 - \zeta_2 + K\beta_1 + J\beta_2 - L(y_3 - y_1) - N^0(y_3 - y_2) \tag{2.5}$$

$$\beta_{32} = \left\{\frac{2}{k_2 - 1}\left[\sigma\left(\frac{k_1 + 1}{2} + \frac{k_1 - 1}{2}\beta_{31}^2\right)^{k_1/(k_1-1)}\right]^{(k_2-1)/k_2} - \frac{1}{\varkappa_2}\right\}^{1/2} \tag{2.6}$$

$$\zeta_3 = \zeta_2 + J(\beta_{32} - \beta_2) + N^\circ(y_3 - y_2)$$

$$n^\circ = \frac{1}{2}\left(\frac{\beta_2 + \zeta_2}{\beta_2\zeta_2 - 1} + \frac{\beta_{32} + \zeta_3}{\beta_{32}\zeta_3 - 1}\right), \quad n_0 = \frac{1}{2}\left(\frac{\beta_0\zeta_0 - 1}{\beta_0 + \zeta_0} + \frac{\beta_1\zeta_1 - 1}{\beta_1 + \zeta_1}\right)$$

$$m = \frac{1}{2}\left(\frac{\beta_1 - \zeta_1}{1 + \beta_1\zeta_1} + \frac{\beta_{31} - \zeta_3}{1 + \beta_{31}\zeta_3}\right) \quad \left(\varkappa_\nu = \frac{k_\nu - 1}{k_\nu + 1},\ \nu = 1, 2\right)$$

$$K = -\frac{1}{k_1 + 1}\left[\frac{\beta_1^2(1 + \zeta_1^2)}{(1 + \beta_1^2)(1 + \varkappa_1\beta_1^2)} + \frac{\beta_{31}^2(1 + \zeta_3^2)}{(1 + \beta_{31}^2)(1 + \varkappa_1\beta_{31}^2)}\right]$$

$$J = -\frac{1}{k_2 + 1}\left[\frac{\beta_2^2(1 + \zeta_2^2)}{(1 + \beta_2^2)(1 + \varkappa_2\beta_2^2)} + \frac{\beta_{32}^2(1 + \zeta_3^2)}{(1 + \beta_{32}^2)(1 + \varkappa_2\beta_{32}^2)}\right]$$

$$L = \frac{1}{2}\left[\frac{\zeta_1(1 + \zeta_1^2)}{y_1(1 + \beta_1\zeta_1)} + \frac{\zeta_3(1 + \zeta_3^2)}{y_3(1 + \beta_{31}\zeta_3)}\right]$$

$$N^\circ = \frac{1}{2}\left[\frac{\zeta_2(1 + \zeta_2^2)}{y_2(\beta_2\zeta_2 - 1)} + \frac{\zeta_3(1 + \zeta_3^2)}{y_3(\beta_3\zeta_3 - 1}\right] \tag{2.7}$$

The (2.2)-(2.7) system is solved by the method of successive approximations. In the first approximation, if we assume that in the coefficient n° the second term equals the first, from (2.2)-(2.3) we can find x_3 and y_3. Moreover, from (2.4) we can find x_1 assuming that in coefficients m and n_0, the second term equals the first, and parameters with a lower index of 1 equal parameters with a lower index of 0. Using the value of x_1, by means of quadratic interpolation we can find y_1, β_1 and ζ_1. Parameters β_{32} and β_{31} are found from Relationships (2.5) and (2.6) which are reduced to a single transcendental equation with a single unknown β_{31}. In the first approximation, the second terms in coefficients K, J, L, N° are assumed to equal the first. Solving the transcendental equation by Newton's method, we find β_{31} and then β_{32} and ζ_3, from Relationships (2.6) and (2.7). In the following iterations, in computing the second terms in the coefficients m, n°, n_0, K, L, J and N, we use the values of $\beta_{31}, \beta_{32}, \zeta_3, y_3, y_1$, ζ_1 and β_1 obtained in the preceding approximation. In an analogous fashion, the corresponding formulas can be found for the remaining three variants.

3. Using the method of characteristics described above, calculations were made of two-layer flow in a nozzle with an angular point. We first considered fan of expansion waves arising through flow around the angular point and then the flow on a given contour. In all of the variants consdiered, it was assumed that $k_1 = 1.14$ and $k_2 = 1.4$ and the ratio between the flow rates in the layers was varied by changing the value of y_* (Figure 1). In all variants the total temperature and total pressure in the two layers were considered equal.

It should be noted that the ratio between the total temperatures does not enter into Relationships (2.2)-(2.7) and in this connection the results obtained may be also used for different total temperature ratios. Thus, the only change is the flow rate ratio between the two layers.

Two-phase axisymmetric supersonic flow was studied in a nozzle with an angular point, designed for uniform, parallel one-layer flow with an output Mach number of $M_0 = 4.6$ and $k = 1.14$. We shall first discuss the results of calculating the parameters in a fan of expansion waves. As a result of the calculations, it was shown that the distribution of M numbers on the nozzle axis for two-layer flows with $k_1 = 1.14$ and $k_2 = 1.4$ differs only slightly from the M number distribution over the axis in one-layer flow with $k = 1.14$ even with flow rates as large as 50% in the boundary layer. As a result of these calculations, it follows that a change of angle θ on the second family characteristics is nonmonotonic and the maximum of angle θ is in the vicinity of the first reflected from the axis of the characteristic CD. It should be noted that similar results were also obtained in calculations of the characteristics of a fan of expansion waves in a one-layer flow, beginning as a uniform flow [3]. The results show that in a fan of expansion waves, at first the M number change along the characteristics of the second family from the line of separation to the axis of symmetry occurs monotonically. However, from the angular point to the line of separation the M number changes nonmonotonically, and a maximum of M in the neighborhood of the characteristic BN is reflected from the line of separation. However, in subsequent characteristics of a fan of expansion waves, the nonmonotonic character of M variation from the angular point to the line of separation occurs only for small gas flow rates through the boundary layer (less than 10%). At large flow rates, M varies monotonically. Naturally, the M number experiences a discontinuity at the line of separation. /80

Let us consider the variation of the flow parameters along the line of separation and compare them with the contour parameters. Figures 3 and 4 show the dependence of the quantities $\zeta = \mathrm{tg}\,\theta$ and $\beta = \sqrt{M^2 - 1}$ on the length on the line of separation and the contour for $y_* = 0.8$, $k_1 = 1.14$, $k_2 = 1.4$. From Figures 3 and 4 and the results, it follows that the values of ζ and β on the line of separation (light dots) and on the contour (dark dots) differ only slightly for $x > 1.5$ even for large flow rates through the boundary layer up to 50%. However, the values of β_{32} and β_{31} on the line of separation differ rather significantly. In Figure 3-6 the radius of the minimum section is taken as the unit of length.

Let us now consider the variation of parameters on the contour for large flow rates in the boundary layer. Figure 5 shows the dependence of the Mach number for one-layer flows (lower curve k = 1.14) and two-layer with $k_1 = 1.14$, $k_2 = 1.4$; $y_* = 0.7$ (upper curve), and with $k_1 = 1.14$, $k_2 = 1.4$, $y_* = 0.8$ (middle curve), from which we can see that the M number for a two-layer flow is greater than the M number for a one-layer flow. Thus, the M number is somewhat larger for

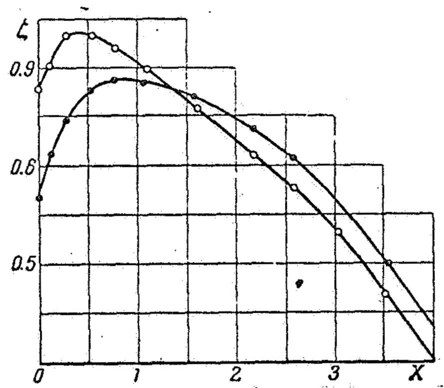

Figure 3.

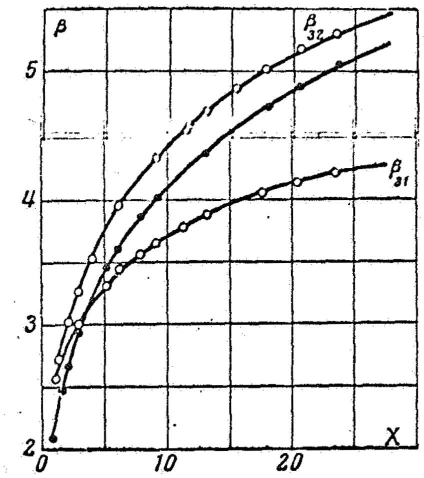

Figure 4.

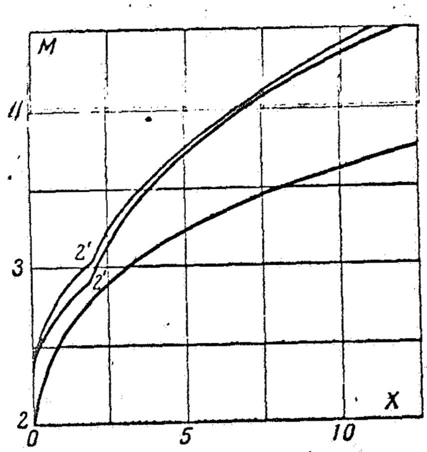

Figure 5.

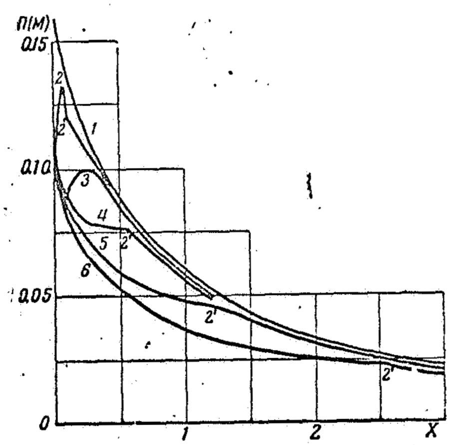

Figure 6.

a thicker boundary layer. The second family characteristics AB (Figure 1) curves away from the line of separation and approaches point N. For boundary layer flow rates greater than 10%, the Mach number in the vicinity of point N varies monotonically. However, at point 2' on the contour there is a discontinuity of the derivative, and — as the flow rate increases through the boundary layer — this point is shifted below the flow. Point 2' on the contour, at which the derivative dM/dx undergoes a discontinuity, is located in the vicinity of the first family of characteristics, curving away from the line of separation and leaving the point of intersection of the line of separation and the last characteristic of the second family.

Figure 6 shows the distribution of pressure along the contour of a nozzle with an angular point for a two-layer flow with $k_1 = 1.14$ and $k_2 = 1.4$ with different values of y_* (i.e. different boundary layer flow rates), as well as the pressure distributions on the same contour in a one-layer flow with k = 1.14. In the figure, curve 1 corresponds to one-layer flow, and the remaining curves — to a two-layer flow, where curves 2, 3, 4, 5, 6 correspond to values $y_* = 0.99, 0.95, 0.9, 0.8, 0.7$. From the figure, it follows that for boundary layer flow rates greater than 10%, the pressure along the nozzle decreases. For flow rates less than 10% in the vicinity of the angular point, there is a positive pressure gradient which arises in the vicinity of point N (Figure 1). From point N the flow in the boundary layer begins to be affected by the internal layer. The occurrence of a positive pressure gradient is connected with the fact that, for one and the same angle of rotation of the flow in Prandtl-Meyer flow, pressure in the flow with a large k decreases more strongly than in a flow with a small k. For relatively large boundary layer thicknesses, the effect of the internal layer begins to appear basically after point 2', so that from point 2' the pressure derivative undergoes a discontinuity and begins to decrease more sharply, while the pressure approaches the pressure of a one-layer flow. After this point, the flow in the boundary layer is basically determined by the internal layer (Figure 6). Up to point 2', turbulence brought about by the internal layer is weakened by the fan of expansion waves. However, at small boundary layer thicknesses, the effect of the internal layer is apparent in the immediate neighborhood of the angular /81 point, and the pressure in the boundary layer strives to equal the pressure in the internal layer. Since the latter is larger with rotation by one and the same angle, a positive pressure gradient arises. The positive pressure gradient increases as the thickness of the boundary layer decreases. Naturally, as the thickness of the layer decreases, the difference between the static pressures in one-layer and two-layer flows at the wall of the nozzle decreases. However, the Mach numbers in such flows then differ considerably. Let us note that the positive pressure gradient in flow around the angular point occurs only when the adiabatic exponent in the boundary layer is greater than the adiabatic exponent in the center of the flow. We should recall that in the work in question, all calculations were made for $\sigma = 1$. For $\sigma \neq 1$, special study is needed on the effects of two-layers on flow around the angular point.

In addition to the calculations for two-layer flows in a nozzle with no angular point using the grid method given in Section 1, special calculations in a nozzle with angular point curvature are also presented. As a result of these calculations, it was shown that with angular point curvature — even with relatively small radii of curvature comprising 0.3-0.5 the radius of the critical section — a positive pressure gradient on the contour of the nozzle does not occur.

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